Solution of Eigen value problems by using New Iterative Method

1. Tahir Kamran
National College of Business Administration and Economics Lahore, (Faisalabad campus)
2 .Dr. M. Jamil Amir
Assistant professor of Mathematics, University of Sargodha, Sargodha (Pakistan)
3. M. Shafeeq Ur Rehman
National College of Business Administration and Economics Lahore, (Faisalabad campus)

Abstract: In this paper the system of ordinary differential equations on Eigen value problems is presented. I used New Iterative Method (NIM) on the Eigen value problems. This is recently developed method which is very easy and efficient developed by Daftardar Gejji and Hossein Jafri [1]. In this study the problems of Eigen value problems are solved to check the ability of this method for solving non linear and linear ordinary differential equations. The results obtained are very useful and close to the exact solution.

Keywords: New iterative method; Eigen value problems; Scientific work place.

1. Introduction

In the large number of problems arising in analysis, mechanics, geometry, and so forth, it is necessary to determine the maximal and minimal of a certain functional. Because of the important role of this subject in science and engineering, considerable attention has been received on this kind of problems. Such problems are called variational problems.

The problem of brachistochrone is proposed in 1696 by Johann Bernoulli which is required to find the line connecting two certain points A and B that do not lie on a vectorial line and possessing the property that a moving particle slides down this line from A to B in the shortest time. This problem was solved by Johann Bernoulli, Jacob Bernoulli, Leibnitz, Newton, and L'Hospital. It is shown that the solution of this problem is a cycloid.

The eigenvalue problems of linear operators are of central importance for all vibration problems of physics and engineering. The vibration of elastic structures, the flutter problems of aerodynamics, the stability problem of electric networks, the atomic and molecular vibrations of particle physics are all the diverse aspects of the same fundamental problem, viz., the principal axis problem of quadratic form.

In view of the central importance of the eigenvalue problem for so many fields of pure and applied mathematics, much thought has been devoted to the designing of efficient methods by which the eigenvalues of a given linear operator may be found. We find the solutions of variational and eigenvalue problems by using the new iterative method. The results are compared with those obtained by the numerical methods available in the literature to establish the efficiency of the method.

2. The New Iterative Method

Consider the following general functional equation

ISSN 2229-5518

$$y(\bar{x}) = f(\bar{x}) + N\big(y(\bar{x})\big), \qquad (1)$$

Where N is nonlinear from a Banach space B→B, f is a known function and $\bar{x} = (x_1, x_2, \dots \dots \dots, x_n)$.

We are looking for a solution y of eq. (1) having the series form

$$y(\bar{x}) = \sum_{n=0}^{\infty} y_i(\bar{x}) \qquad (2)$$

The nonlinear operator N can be decomposed as

$$N\left(\sum_{n=0}^{\infty} y_i\right) = N(y_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\} \qquad (3)$$

From Equations (2) and (3), Eq. (1) is equivalent to

$$\sum_{i=0}^{\infty} y_i = f + N(y_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\}. \qquad (4)$$

We define the recurrence relation

$$y_0 = f,$$

$$y_1 = N(y_0), \qquad (5)$$

$$y_{m+1} = N(y_0 + \dots + y_m) - N(y_0 + \dots + y_{m-1})$$

Then

$$(y_1 + \dots + y_{m+1}) = N(y_0 + \dots + y_m),$$

and

$$\sum_{i=0}^{\infty} y_i = f + N\left(\sum_{i=0}^{\infty} y_i\right). \qquad (6)$$

The k-term approximation solution of Eq. (1) is given by $y = y_0 + y_1 + \dots + y_{k-1}$

If N contracts i.e.$\| N(x) - N(y) \| \leq k \| x - y \|$, $0 < k < 1$, then

$$\| y_{m+1} \| = \| N(y_0 + \dots + ys_m \| - N \| y_0 + \dots + y_{m-1} \| \leq k \| y_m \| \leq k^m \| y_0 \|,$$

m =0, 1, 2……

And series $\sum_{i=0}^{\infty} y_i$uniformly and absolutely converges to solution of equation (1). A unique solution, with respect to Banach fixed point theorem [15].

3. The New Iterative Method and its applications on and Eigen value problems

ISSN 2229-5518

Example 1: Consider the following Eigen value problem

$$y'' + \lambda y = 0$$

with boundary conditions $y(a) = y(-a), \quad y'(a) = y'(-a)$

This is the corresponding integral equation.

$$y(x) = B + Ax - \lambda \int_0^x \int_0^x y\,dx\,dx$$

Setting $y_0 = B + Ax, \; N(y) = -\lambda \int_0^x \int_0^x y\,dx\,dx$

Using condition $y(-a) = y(a)$, we have $A = 0$ so that $y^0 = B$.

Applying the algorithm of new iterative method the successive iterations are

$$y_1 = N(y_0) = -\left(\frac{1}{2}\right) Bx^2\lambda$$

$$y_2 = \left(\frac{1}{24}\right) Bx^4\lambda^2$$

$$y_3 = -\left(\frac{1}{720}\right) Bx^6\lambda^3.$$

The four term approximate solution is given by

$$y(x) = y_0 + y_1 + y_2 + y_3 = -\left(\frac{1}{720}\right) Bx^6\lambda^3 + \left(\frac{1}{24}\right) Bx^4\lambda^2 - \left(\frac{1}{2}\right) Bx^2\lambda + B.$$

Using the boundary condition $y'(a) = y'(-a)$, the eigen value is

$$\lambda_{\{n\}} = \left(\left(\frac{n\pi}{a}\right)\right)^2, n = 0, 1, 2, \ldots.$$

Example .2: Consider the following eigen value problem

$$y^4 + \lambda y'' = 0$$

with boundary conditions $y(0) = 0,\ y''(0) = 0, y(1) = 0\ , y'(1) = 0.$

This is the corresponding integral equation.

$$y(x) = Bx + A\frac{x^3}{6} - \lambda\int_0^x\int_0^x\int_0^x\int_0^x y'' dx\, dx dx\, dx.$$

Setting $y_0 = Bx + A\frac{x^3}{6}$, $N(y) = -\lambda\int_0^x\int_0^x\int_0^x\int_0^x y'' dx\, dx dx\, dx$

and applying the algorithm of new iterative method we obtain the successive iterations as

$$y_1 = N(y_0) = N(y_0) = -\left(\frac{1}{120}\right)Ax^5\lambda$$

$$y_2 = \left(\frac{1}{5040}\right)Ax^7\lambda^2$$

$$y_3 = -\left(\frac{1}{362880}\right)Ax^9\lambda^3.$$

The four term approximate solution is

$$y(x) = y_0 + y_1 + y_2 + y_3 = -\left(\frac{1}{362880}\right)Ax^9\lambda^3 + \left(\frac{1}{5040}\right)Ax^7\lambda^2 - \left(\frac{1}{120}\right)Ax^5\lambda + \left(\frac{1}{6}\right)Ax^3 + Bx$$

$$\Rightarrow y(x) = Bx - Ax + Ax - \left(\frac{1}{362880}\right)Ax^9\lambda^3 + \left(\frac{1}{5040}\right)Ax^7\lambda^2 - \left(\frac{1}{120}\right)Ax^5\lambda + \left(\frac{1}{6}\right)Ax^3.$$

Using the given boundary conditions we obtain the eigen value after some calculations as

$$tan(\sqrt{\lambda}) = \sqrt{\lambda}.$$

So using the iterative method the zeros of this equation in the interval (0,20) are

$$0, 4.4934, 7.7253, 10.9041, 14.0662, 17.2208,$$

Example 3: Consider the following eigen value problem

$$y'' + \lambda y = 0$$

with boundary conditions $y(0) = 0$, $y, y^{/}$ $are\ finite\ as\ x \to \infty.$

ISSN 2229-5518

This is the corresponding integral equation.

$$y(x) = Ax - \lambda \int_0^x \int_0^x y dx\, dx$$

Following our algorithm we set $y_0 = Ax$ $N(y) = -\lambda \int_0^x \int_0^x y dx dx$.

The successive iterations are

$$y_1 = N(y_0) = -\left(\frac{1}{6}\right) A x^3 \lambda$$

$$y_2 = \left(\frac{1}{120}\right) A x^5 \lambda^2$$

$$y_3 = -\left(\frac{1}{5040}\right) A x^7 \lambda^3$$

$$y_4 = \left(\frac{1}{362880}\right) A x^9 \lambda^4$$

$$y^5 = -\left(\frac{1}{39916800}\right) A x^{11} \lambda^5.$$

The six term approximate solution is

$$y(x) = y_0 + y_1 + y_2 + y_3 + y_4 + y_5$$

$$= -\left(\frac{1}{39916800}\right) A x^{11} \lambda^5 + \left(\frac{1}{362880}\right) A x^9 \lambda^4 - \left(\frac{1}{5040}\right) A x^7 \lambda^3 + \left(\frac{1}{120}\right) A x^5 \lambda^2 - \left(\frac{1}{6}\right) A x^3 \lambda + Ax$$

$$\Rightarrow y(x) = \left(\frac{A}{\sqrt{\lambda}}\right)\left(-\left(\frac{1}{39916800}\right) x^{11} \lambda^{\frac{11}{2}} + \left(\frac{1}{362880}\right) x^9 \lambda^{\frac{9}{2}} - \left(\frac{1}{5040}\right) x^7 \lambda^{\frac{7}{2}} + \left(\frac{1}{120}\right) x^5 \lambda^{\frac{5}{2}} - \left(\frac{1}{6}\right) x^3 \lambda^{\frac{3}{2}} + x\right)$$

$$\Rightarrow y(x) = \left(\frac{A}{\sqrt{\lambda}}\right)(sin\sqrt{\lambda} x)$$

ISSN 2229-5518

where $A \neq 0$. Since $y, y^{/}$ are finite as $x \to \infty$ therefore $\boldsymbol{\lambda}$ can assume any positive real value.

Acknowledgement

The authors are very grateful to Assistant Prof. Mr. M. Yaseen for his guidance about the NIM and helping him to learn software. The first author is also thankful to Assistant Prof. Dr.M. Jamil Amir for his guidance and support.

4.3 Conclusion

In this paper, I have discussed the eigen value problems using recently developed new iterative method developed by Versha Daftardar Gejji and Hossein jafari. The results obtained are very close to the exact solution and the calculations are also reduced. Using few iterations we obtained the desired results. These results show the efficiency and effectiveness of the new iterative method.

REFERENCES

[1] Varsha Daftardar-Gejji, Hossein Jafri,(2006) An iterative method for solving nonlinear functional equations, *J. Math. Appl*. 316, 753–763.

[2] Sachin Bhalekar,Varsha Daftardar-Gejji,(2008) New iterative method: Application to partial differntial equations, Appl. Math. Comput. 203, 778–783.

[3] V. Daftardar-Gejji and S. Bhalekar,(2010) Solving fractional boundary value problems with Dirichlet boundary conditions using a new iterative method, Computers & Mathematics with Applications 59 , 1801–1809.

[4] V. Daftardar-Gejji and S. Bhalekar,(2008) An Iterative method for solving fractional differential equations, *Proceedings in Applied Mathematics and Mechanics* 7, 2050017–050018.

[5] S. Bhalekar and V. Daftardar-Gejji,(2010) Solving evolution equations using a new iterative method, *Numerical Methods for Partial Differential Equations* 26 , 906–916.

[6] M. Yaseen and M. Samraiz,(2012) A modified new iterative method for solving linear and nonlinear Klein-Gordon Equations, *Appl. Math. Sci. 6 , 2979–2987.*

[7] G. Adomian,(1986) A new approach to the heat equation—an application of the decomposition method, *Journal of Mathematical Analysis and Applications 113 , no. 1, 202–209.*

[8] G. Adomian,(1988) A review of the decomposition method in applied mathematics, *Journal of Mathematical Analysis and Applications 135 , no. 2, 501–544.*

[9] G. Adomian,(1991) Solving frontier problems modelled by nonlinear partial differential equations, *Computers & Mathematics with Applications 22 (1991), no. 8, 91–94.*

[10] Kluwer Academic, Dordrecht,(1994) Solving Frontier Problems of Physics: The Decomposition Method, *Fundamental Theories of Physics, vol. 60*

[11] Mehdi Dehghan and Mehdi Tatari,(2006) The use of Adomian decomposition method for solving problems in calculus of variations, *Math. Prob. Eng. 1(2006), 1-12.*

[12] Y. Saad,(1992) Numerical Methods for Large Eigenvalue Problems, *Halsted Press, Div. of John Wiley & Sons, Inc., New York.*

[13] Lloyd N. Trefethen and Mark Embree,(2005) Spectra and Pseudospectra, Princeton University Press, Princeton.

[14] Chao-Kuang Chen and Shing-Huei ho, (1996) Application of differential Transformation to EigenValue Problem, Applied Mathematics and Computation 79: 173-188(1996).

[15] A.J.Jerri, (1999) Introduction to Integral Equations, *Second edition, Wiley-Interscience.*

[16] A. wazwaz,(1997) A first coursein Integral Equations, *World Scientific*.

[17] A.Wazwaz,(1999) A relieable modification of Adomian decomposition method, *Applied Math. Comput. 102, 77-86*

[18] R.Z.Ouedraogo, Y.Cherrault, K.Abbaoui,(2000) Convergence of Adomian's decomposition method applied to algebraic equations, *Kybernetes 29, 1298-1305.*

[19] H.Jafari,V.Daftardar-gejji, Revised Adomian decomposition method for solving a system of nonlinear equations, *submitted for publication*.

[20] E.Babolian, J.Biazar, A.R.Vahidi,(2004) Solution of a system of nonlinear equations by Adomian decomposition method, *j. Math. Annal. Appl. 150, 847-854*

[21] I. Podlubny,(1999) fractional Differential Equations, *Academic press*.